10%

Large Sample test based on Normal distribution ($n > 30$).

* Single mean
* Difference between means

Single mean :-

If σ is known then $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

If σ is unknown then $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

where $\bar{x}$ - Sample mean.

μ - population mean

σ - population SD

s - Sample SD and n - no. of samples

* **Problems:**

A sample of 900 members has a mean 3.4 cm and Standard deviation 2.61cm. Is the sample from a large population of mean 3.25 cms and SD of 2.61 cms? (Test at 5% LOS). The value of z at 5% level is $|z_\alpha| < 1.96$).

Soln:

$n = 900, \ \bar{x} = 3.4, \ s = 2.61$

$\mu = 3.4, \ \sigma = 2.61$

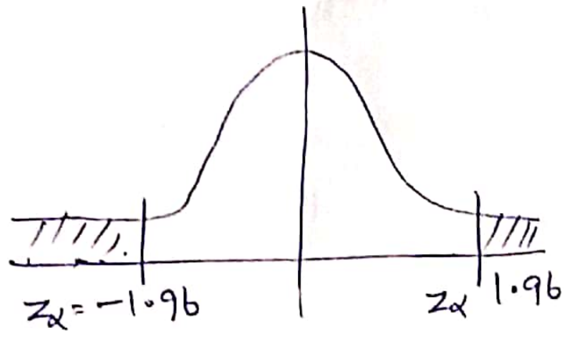
$\alpha = 5\%$

1. $H_0 : \mu = 3.25$

2. $H_1 : \mu \neq 3.25$ (Two-tailed test)

3. $\alpha = 5\%$

4. Critical region



5. Test Statistic

$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.4 - 3.25}{\left(\frac{2.61}{\sqrt{900}}\right)} = 1.724$$

6. Conclusion :

At 5% LOS critical value $Z_\alpha = 1.96$.

compare $|Z|$ & Z_α

$|Z| = 1.724 < Z_\alpha = 1.96$

Hence we accept H_0.

The mean breaking strength of the cables supplied by a manufacturer is 1800 with a S.D of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. In order to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS?

Soln:

$n = 50$, $\bar{x} = 1850$ $\mu = 1800$, $\sigma = 100$.

1. $H_0 : \mu = 1800$

2. $H_1 : \mu > 1800$ [use one-tailed test (right)]

3. LOS $\alpha = 1\%$

4. Critical value z_α at $1\% = 2.33$ — Table value

5. test statistic

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}}$$

$$= 3.54$$

6. conclusion

Calculated 'z' value $|z| \ngtr z_\alpha$ ie, $3.54 > 2.33$

Reject Ho which means accept H_1.

ie, we may support the claim of accept increasing in breaking strength.

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3. A normal population has a mean of 6.48 and s.d of 1.5. In a sample of 400 members mean is 6.75. Is the difference significant?

Solution:

$n = 400$, $\mu = 6.48$, $\sigma = 1.5$ and $\bar{x} = 6.75$.

1. $H_0: \mu = \bar{x}$ (no significant difference)

2. H_1: $H_1: \mu \neq \bar{x}$

3. Los $\alpha = 5\%$

4. Table value Z_α at 5% Los = 1.96

5. $$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{6.75 - 6.48}{\frac{1.5}{\sqrt{400}}} = 3.6$$

6. Conclusion :-

$|Z| \nless Z_\alpha$

We reject Ho.
Hence the difference is significant.

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4. The average number of defective articles per day in a certain factory is claimed to be less than the average of all the factories. The average of all the factories is 30.5. A random sample of 100 days showed the mean value as 28.8 and SD showed as 6.35. Is the average less than the figure ~~of~~ for all the factories?

Soln: $n = 100$, $\mu = 30.5$, $s = 6.35$, $\bar{x} = 28.8$

1. $H_0 : \mu = 30.5$

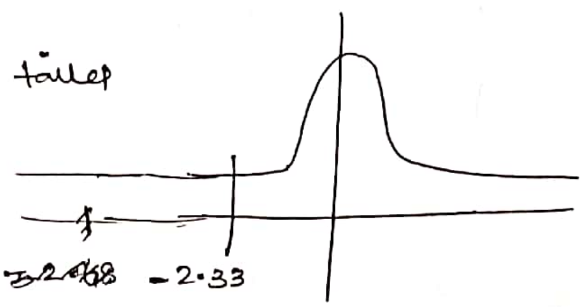
2. $H_1 : \mu < 30.5$ (left tailed test)

3. $\alpha = 1\%$

4. Z_α at 1% is 2.33

5. $$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{28.8 - 30.5}{\left(\frac{6.35}{\sqrt{100}}\right)} = -2.68$$

Left tailed



Conclusion:

$|z| = 2.68$

$|z| \not< 2.33$

We reject Ho. So we accept H_1.

Hence the average less than the figure for all the factories.

5. The mean life time of a sample of 100 light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hrs, against the alternative hypothesis $\mu \neq 1600$ hrs with $\alpha = 0.05$ & 0.01.

Soln.

$n = 100, \quad \mu = 1600, \quad s = 120, \quad \bar{x} = 1570$

$\alpha = 0.05$ and $\alpha = 0.01$

1. $H_0 : \mu = 1600$
2. $H_1 : \mu \neq 1600$
3. (i) $\alpha = 5\%$ (ii) $\alpha = 1\%$
4. $Z_\alpha = 1.96$ $Z_\alpha = 2.58$
5. Test statistic $z = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

$$z = \frac{1570 - 1600}{\left(\frac{120}{\sqrt{100}}\right)} = -2.5$$

6. Conclusion

(i) $|z| = 2.5 \neq$ Table value at 5%.

We reject Ho.

(ii) $|z| = 2.5 <$ Table value at 1%.

We accept Ho.

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Large Sample test based on Normal distribution for difference of means.

If σ is not known,

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim N(0,1)$$

where $\bar{x}_1$ – mean of the first sample

$\bar{x}_2$ – mean of the second sample

s_1 – SD of first sample

s_2 – SD of second sample.

If σ is known

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

σ_1 – SD of population.

Working procedure :-

1. Null Hypothesis $H_0 : \mu_1 = \mu_2$
2. Alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ Two tailed

   $H_1 : \mu_1 > \mu_2$ Right tailed

   $H_1 : \mu_1 < \mu_2$ Left tailed
3. LOS
4. Z_α
5. $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad (\text{or}) \quad z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$
6. Conclusion $|z| < Z_\alpha$ accept H_0

   otherwise Reject H_0.

Problems:

1. The means of two large samples of 1000 & 2000 members are 67.5 inches and 68.0 inches respy. Can the samples be regarded as drawn from the same population of SD 2.5 inches?

Soln:

$n_1 = 1000, \quad n_2 = 2000$

$\bar{x}_1 = 67.5 \quad \bar{x}_2 = 68$

$\sigma_1 = \sigma_2 = \sigma = 2.5$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$ (two tailed)
3. LoS $\alpha = 5\%$
4. $Z_\alpha = 1.96$
5. $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{67.5 - 68}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{2000}}}$$

$$= \frac{-0.5}{\sqrt{0.00625 + 0.003125}} = -5.16$$

Conclusion

$|z| = 5.16$

Cal z $|z| \nless 1.96$ (Table value)

We reject H_0.

2. Random samples drawn from two countries give the following data relating to the heights of adult males. Is the difference b/w SD significant ?

	Country A	Country B
Mean height (in inches)	67.42	67.25
SD (in inches)	2.58	2.50
Number in samples	1000	1200

Soln:

$n_1 = 1000$, $n_2 = 1200$

$\bar{x}_1 = 67.42$ $\bar{x}_2 = 67.25$

$s_1 = 2.58$ $s_2 = 2.50$

1. $H_0 : \mu_1 = \mu_2$

2. $H_1 : \mu_1 \neq \mu_2$ (Two tailed)

3. $\alpha = 5\%$

4. $Z_\alpha = 1.96$

5. $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{67.42 - 67.25}{\sqrt{\frac{(2.58)^2}{1000} + \frac{(2.50)^2}{1200}}}$$

$$= \frac{0.17}{0.1085} = 1.567$$

6. Conclusion $|Z| = 1.567$

~~Tab~~ Calulate Z < Table value of Z

We accept H_0.